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Natural Science and Mereology

1. Mereological sciences. Claims which belong to formal mereology are often (just as in geometry) illustrated by means of concrete examples and the border between formal and applied mereology is not explicitly discussed. In this article, however, the distinction between formal mereology and applied mereology will be strictly adhered to. Formal mereology is, like mathematics and traditional formal logic, a purely formal discipline. It tells us in itself nothing about particular things, events, and processes in the spatiotemporal world. However, its central notion, ‘part’ (P) or ‘proper part’ (PP), contains more than merely syntactical content in contradistinction to the central notions of formal logic, the logical constants. In this sense, mereology seems to be more akin to mathematics than to formal logic.

In science we have had for a long time both mathematics and mathematical physics, but there is so far no mereological physics, and nor is there any other kind of empirical-scientific discipline in which mereology has become an integral part. If it is on its way to enter any discipline in a thorough-going way, it is probably informatics. In computer ontologies such as the Foundational Model of Anatomy (FMA) and the Gene Ontology (GO), the parthood relation plays – for natural reasons – a prominent role.

This article will relate formal mereology to parthood relations used in the natural sciences. But general relativity, quantum mechanics, and string theory will be completely left out of account. These theories contain, each in its own way, peculiar part-whole relations that cannot

be discussed here; witness notions such as ‘spatiotemporal nonseparability’, ‘state nonseparability’, and ‘feature nonseparability’ (Healey 2004).

2. Parthood and proper parthood. In formal mereology it does not matter whether PP is defined by means of P or vice versa. But in the empirical sciences PP is always the natural point of departure. Why? Because it is often an empirical question what kind of proper parts a certain kind of whole has, but the fact that a whole is identical with itself is a logical truth. In making ‘PP’ and identity (‘=’) primitive terms and ‘P’ a defined term, ‘ $P_{xy} =_{df} (PP_{xy} \vee x=y)$ ’, the epistemological difference that in applied mereology exists between ‘PP’ and ‘=’ is not blurred. Therefore, in what follows, only the relation of proper parthood will be talked about.

3. Formal and applied mereology. The fact that physics has become mathematical physics does not mean that physics and mathematics have become identical, or that physics has become a sub-discipline of mathematics. Mathematics is still a formal science and physics an empirical science, although one where mathematics can be applied. In the world studied by physics and chemistry there are no *purely* mathematical entities. There are no numbers, for example, but rather things such as 1 oxygen atom, 2 oxygen atoms, 3 oxygen atoms, etc., a thing of 1 m length, a thing of 2 m length, a thing of 3 m length, etc., a thing of 1 kg mass, a thing of 2 kg mass, a thing of 3 kg mass, etc. And so on with respect to physical and chemical kinds of entities as well as to kinds of physical dimensions. But by means of the numerals ‘1’, ‘2’, ‘3’ in isolation it is impossible to refer to any pure numbers 1, 2, 3 that are situated in space and time: if one is asked to point to, for instance, the mathematical number 2, one does simply not know what to point to. Before arithmetic can enter the disciplines of physics and chemistry, numerals have to be connected to count nouns (‘oxygen atom’) or something that functions like count nouns (‘meter unit’, ‘kilogram unit’).

Arithmetic deals with *numbers*, but mathematical physics deals with *quantities*. Whatever the true ontological status of different kinds of mathematical numbers is, no such numbers

have any direct instances or exemplifications in the physical-chemical world. Pure numbers are thus abstract, in the sense that they are not spatiotemporally localizable.

What has just been said of the relation between mathematics and mathematical physics is also true for the relation between formal and applied mereology. If one is asked to point to the *pure* formal-mereological PP-relation, one does not know what to point to; as soon as one points to a part, it is a part of some sort. This is connected with the fact that axiomatic mereology is a formal discipline. Every parthood relation in the world is a parthood relation of some sort. For instance, ordinary things have *spatial* parts, processes have *temporal* parts, and organisms and machines have *functional* parts; none of them has just *parts*. As numerals have to be connected to count nouns in order to be able to refer to something in the world, so the PP-predicate of formal mereology has to be connected to adjectives such as ‘spatial’, ‘temporal’, and ‘functional’.

Below, four kinds of non-formal parthood relations will be distinguished and commented on: pure spatiotemporal parthood, physical-chemical spatial parthood, functional parthood, and constitutive parthood. They will be related to the formal system of Minimal Mereology, i.e., to a system defined by the four axioms of irreflexivity (A1), asymmetry (A2), transitivity (A3), and weak supplementation (A4: if a whole has a proper part, it has also another proper part that is disjoint from the first). First, however, some words about one kind of natural-scientific entity that conforms to A1-A3 but not necessarily to A4: scales.

4. Scales. Both mere ordinal scales (such as Mohs’ scale for mineral hardness and Beaufort’s scale for wind force) and metric scales (such as those for length, mass, and temperature) are strict partial orderings, and could in this sense lay claim to conform to A1-A3. Some but not all of them may also lay claim to conform to A4 (see 4a below). But even though the relations larger-smaller and higher-lower might seem to be analogous to the

whole-part relation, they are in metrological and measurement literature hardly ever spoken of as containing parthood relations; and there are good reasons for this (4b below).

(4a) Applied to a scale, axiom A4 says that if one determinate value of the scale has another determinate (smaller/lower) value as a proper part, then there must be still another value (= proper part) that lies between them. The axiom is true for all continuous metric scales, since between any two values on a continuous scale there is necessarily a third value; if, for example, the value '18 kg' is regarded as a whole that has the value '17.9999' kg as a proper part, then there are proper parts/values between these two values. But A4 is false for all discontinuous metric scales and ordinal scales; between the value 'strong breeze' (whole) of the Beaufort scale and the next lower value, 'fresh breeze' (proper part), there is no other value (proper part) at all.

(4b) Scales such as those for length and mass have a non-conventional zero point, whereas others such as the Celsius and Fahrenheit scales for temperature have a conventionally chosen zero point. In the former cases, the analogy between larger-smaller and whole-part works to some extent, but in the latter cases it breaks down. It would be odd to regard 'minus one degree Celsius' to be a proper part of 'zero degree Celsius'.

5. Pure spatiotemporal parthood. Pure spatial proper parthood conforms unproblematically to the four axioms of Minimal Mereology. If circular time is disregarded, the same is true of pure temporal proper parthood; in circular time, one turn seems to both a proper part and the whole of time. It should be noted, though, that a relation such as 'being a *large* spatial part' does not conform to the transitivity axiom (Johansson 2004). That is: x can be a large spatial part of y , and y can be a large spatial part of z , but yet x need not necessarily be a large spatial part of z . This is probably due to the fact that 'large' is a so-called 'logically *attributive* adjective', i.e., the predication in question cannot be split up into two self-sufficient predications: 'is a large part' and 'is a spatial part'; to be a large spatial part is to be

large *as* a spatial part. (The adjective ‘blue-colored’ in ‘being a blue-colored spatial part of a multi-colored surface’ is, in contrast, a ‘logically *predicative* adjective’; to be a blue-colored spatial part is here to be a blue-colored part *and* a spatial part.)

6. Physical-chemical spatial parthood. The parthood relation of formal mereology, PP_{xy}, takes as its arguments individuals. From a non-nominalistic ontological point of view, there are two kinds of individuals: *concrete* (i.e., spatiotemporally localizable entities such as ordinary things and instances of natural kinds and properties) and *abstract* (i.e., not spatiotemporally localizable entities such as propositions, numbers, sets, and universals).

Physical-chemical spatial parthood relations between concrete individuals conform without further ado to the axioms of Minimal Mereology, but some such relations between abstract individuals require a distinction between two kinds of parthood relations. They can be called ‘is part of’ (or ‘part-to-whole parthood’) and ‘has as part’ (or ‘whole-to-part parthood’), respectively (Smith 2008, p. 112).

If the natural kinds *water molecule* and *oxygen atom* are regarded as two so-called ‘structural universals’ (or two sets of natural kind instances), then one cannot truly write simply ‘PP(oxygen atom)(water molecule)’. Why? Because even though it is true that ‘*water molecule* has *oxygen atom* as part’ it is false that ‘*oxygen atom* is part of *water molecule*’. In other words, it is true that each instance of a water molecule has an oxygen atom as a part, but false that each instance of an oxygen atom is part of a water molecule; there are oxygen atoms outside of water molecules. Similarly, but conversely, it is true that ‘*human eye* is part of *human organism*’, since so far there are no eye transplants, but false that ‘*human organism* has *human eye* as part’, since there are people who have lost an eye.

Both ‘part-to-whole parthood’ and ‘whole-to-part parthood’ conform to the axioms of Minimal Mereology, but they must be kept distinct when the relata are structural universals (or corresponding sets of instances). When the relata are two spatiotemporal individuals, it

holds true: ‘part-to-whole parthood’ if and only if ‘whole-to-part parthood’. And when the relata are sets of individuals, S_1 and S_2 , the following can be the case: some but not all members of S_1 have a part-to-whole relation to members of S_2 , and some but not all members of S_2 have a whole-to-part relation to members of S_1 . For instance, some members of the set of pig hearts (S_1) have been transplanted into human organisms, and become parts of members of the set (S_2) of such organisms (Johansson 2008).

7. Functional parthood. Some parthood relations talked about in the natural sciences seem not to conform to the transitivity axiom (A3) of formal mereology. Two examples: (i) a cell’s nucleus is (a functional) part of a cell, and the cell can be (a functional) part of an organ, but yet the nucleus is not (a functional) part of an organ; (ii) heart cells are (functional) parts of the heart, and the heart is (a functional) part of the circulatory system, but yet the cells are not (functional) parts of the circulatory system.

Starting with (Rescher 1955), there has been some discussion in mereology about these and similar examples from other areas. What many of them have in common is that the parthood relation talked about can be subsumed under the label ‘direct functional parthood’, i.e., the expression ‘is part of’ should be read as meaning ‘is direct functional part of’, which is an expression that belongs to applied mereology. A heart cell’s nucleus is a direct functional part of the cell in the sense that it contributes directly to the cell’s functioning, but it is only an *indirect* functional part of the heart in the sense that it contributes only indirectly to the heart’s functioning.

In relation to this transitivity problem, the mainstream view among mereologists is that “if the general intended interpretation of ‘part’ is narrowed by additional conditions (e.g., by requiring that parts make a direct contribution to the functioning of the whole), then obviously transitivity may fail. In general, if x is a φ -part of y , and y is a φ -part of z , x need not be a φ -part of z : the predicate modifier ‘ φ ’ may not distribute over parthood. But this shows the

non-transitivity of ‘ ϕ -part’ (e.g., of *direct* part, or *functional* part), not of ‘part’. And within a sufficiently general framework this can easily be expressed with the help of explicit predicate modifiers” (Varzi 2003).

One criticism of this view claims that there is no “sufficiently general framework” of mereology that can handle these cases. It is claimed that, implicitly, direct functional parthood is at least a three-term relation, whereas by definition formal mereology is concerned only with a two-term parthood relation; and no predicate modifiers will be able to change this fact (Johansson 2006a). When, for instance, it is said that the nucleus (first relatum) is a direct functional part of the cell (second relatum), it is also implicitly said that this nucleus has a causal relation to at least one other proper part (third relatum) of the cell.

The predication ‘*x* is a direct functional part’ cannot be split up into ‘*x* is a direct part’ and ‘*x* is a functional part’; to be a direct functional part is to be direct *as* a functional part. Therefore, in conformity with what was earlier (section 5) said about ‘large spatial part’, one might also say that ‘direct functional part’ involves a logically *attributive* adjective, and that contemporary formal mereology seems not to be able to handle predications with such terms.

8. Constitutive parthood. The relation of constitution differs from identity. To claim ‘E is constituted by F’ is not to claim that E and F are identical. The oxygen atom is a physical-chemical part of the water molecule; also, it is part of a unity that consists of itself, two hydrogen atoms, and the force relations between them. If the water molecule has so-called emergent causal powers, then the molecule cannot be identical with this unity, only constituted by it.

Constitution is irreflexive (A1) and asymmetric (A2). Some take it also to be transitive and to conform to A3 (Simons 1987, p. 237), whereas others take it to be a non-transitive and non-mereological relation (Baker 2000, p. 44; 2007, p. 181). Probably, this difference has to do with whether ‘constitution’ is understood in a wide sense or in the narrow sense of *direct*

constitution, which makes it non-transitive (compare the remarks about ‘direct functional part’ in section 7). Here, constitution will only be discussed in the light of axiom A4 of Minimal Mereology (if a whole has a proper part, it has also another proper part that is disjoint from the first).

Often, the relation of constitution is used in order to explain how certain kinds of entities, e.g., organisms, can retain their identity through time in spite of exchanging (constituting) proper parts. For the purposes of this article, however, it is enough to discuss constitution at a certain time. In order to characterize such synchronic constitution, it is commonly said that the constituted whole and the sum of the constituting parts, despite not being identical, must nonetheless coincide in space; this point can also be made by saying that the whole is more than the sum of its constituting parts. The term ‘sum’ now used is informal, and must not be conflated with that of ‘mereological sum’.

The most conspicuous examples of constitution are to be found in the so-called ‘Gestalten’ of perceptual psychology (a discipline that is sometimes regarded as belonging to natural science). The well-known face of the computer smiley, :-), is such a Gestalt; no doubt, it contains in some sense the four elements here depicted

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plus their specific spatial relations. But when a smiley is perceived, more is perceived than the elements in their spatial relations, there is also – at the same place – the smiley Gestalt.

Without the unity of the-elements-in-their-spatial-relations (C) there would be no smiley (W), but the latter whole (W) is nonetheless not identical with the unity (C). This being noted, axiom A4 can enter the stage. It seems natural to say that C is a proper part of W, if only an underlying constituting part, but in order for the constitution relation to be a mereological relation, there has to be – according to A4 – a proper part beside C, i.e., a proper part that is not a constitutive part. Since according to the assumptions there is no proper part that is

spatially distinct from C, if there is such a further proper part, it must in some sense of ‘supervenience’ be a supervening entity (S); call it a ‘smiling-face-meaning’. If such an S exists, then the whole smiley is equal to the sum of *all* its parts ($W = S + C$), and conforms to A4, even though it is more than the sum of its *constituting* parts (Johansson 2006b).

8. State of discussion. The problem here presented, what the relation between formal mereology and theories in the natural sciences look like, has so far received very little attention.

Further Readings

Simons, P. M., 1987, *Parts. A Study in Ontology*, Oxford: Clarendon Press. This classic mereological treatise, unlike most other contemporary ones, makes ‘proper part’, not ‘part’, a primitive term; and it discusses both ‘constitution’ (Chapter 6.5) and ‘integral wholes’ (Chapter 9).

Casati, R. and Varzi, A. C., 1994, *Holes and Other Superficialities*; 1999, *Parts and Places: The Structures of Spatial Representation*, both from Cambridge (MA): MIT Press. These two books can be regarded as the first thorough-going attempts to create an applied mereology; in Chapter 3.2 of 1999 the transitivity problem is discussed.

Varzi, A. C., 2006, “A note on the transitivity of parthood,” *Applied Ontology* 1, 141-146; and Johansson, I., 2006a, “Formal mereology and ordinary language – Reply to Varzi,” *Applied Ontology* 1, 157-161; here the transitivity problem and that of the relation between formal and applied mereology is discussed.

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