(Published in *Applied Ontology*, Volume 1, Number 2, 2005/2006, pp. 157-161)

Formal Mereology and Ordinary Language – Reply to Varzi

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Achille Varzi ends his note [10] on my paper [6] by saying: "Parthood is transitive, ϕ-parthood—for many values of 'ϕ'—is not." Let me call this view 'the (Simons/Casati/Varzi) predicate modifier account' of non-transitive parthood relations [9, 2]. As summarized in this quotation, I agree with it here, and did agree with it in the paper Varzi criticizes. It is puzzling to me why he surrounds his specific comments on subsumption, relative products, and functions with statements such as "we end up with a reinforcement of the standard defense of transitivity on behalf of classical mereology" ([10], abstract). I have not questioned this transitivity, and until reading Varzi's reply thought that I had made this perfectly clear in my paper. Rather, my thoughts about the intransitivity examples have led me to the view that they bring sharply to the fore a more general problem: *what does the relation between classical formal mereology and the parthood predicates used in science and ordinary language look like*. My paper ends with "a warning: be careful if you try to apply the transitivity axiom of binary mereology to parthood predicates found in areas outside mereology proper. Such predicates might very well be intransitive, non-transitive or fall outside the scope of any natural definition of transitivity ([6], p. 180)."

Perhaps an analogy can illuminate what my warning amounts to. In undergraduate courses in logic, students are taught how to transform sentences in ordinary language into sentences in first order predicate logic. Sometimes this can be done smoothly, but sometimes it is hard and sometimes not even possible. In my opinion, when mereology is applied in ontological engineering, similar situations can obtain: *as formalizations of ordinary language can be problematic, "mereologizations" of ordinary language can be problematic.*

Since computer and information scientists have to relate themselves to pre-existing predicates that in ordinary life are used as means to talk about the world, I think this view of mine is quite relevant to what has given this journal its name: *Applied Ontology*. It is good for authors and developers of all domain-specific computer ontologies such as the Gene Ontology (GO) [4] and the Foundational Model of Anatomy (FMA) [3], which contain (proper) parthood relations, to know (i) whether "their" parthood relations (which I think are ϕ-parthoods) are in general transitive or not, and, (ii) if not, whether these relations are nonetheless transitive in some specific area that is important for the practical purposes at hand.

Now to the more specific issues in Varzi's paper (note: the section numbers below do not mirror Varzi's).

1. When I first read about Varzi's views on ϕ-parthoods (in [2]), I understood his claim to be that ϕ-parts are to parts what "ϕ-mammals" such as cats and horses are to mammals. That is, I took him to be implicitly using the traditional philosophical class subsumption relation, but he isn't. The fact that cats are in the traditional sense subsumed under mammals does not only mean that the set of cats is a subset of the set of mammals, it also means that cats have all the features that mammals have. To Varzi, however, 'x is subsumed by y' is more or less synonymous with the notion ‘x has a smaller extension than y’, where the relata can be not just “natural classes” but any kinds of entities whatsoever. But I see no need to quarrel about Varzi’s definition of ‘subsumption’. Terms can be given many definitions, and I am not a language purist. It is often hard to see whether people have good or bad reasons for using old terms in new ways. I would, though, have been more happy if Varzi had simply retorted that he never intended the relation between ‘ϕ-part’ and ‘part’ to be one of traditional subsumption. However, now I understand how he uses his terms and looks upon the relation between ‘ϕ-part’ and ‘part’, and I have no longer any “charge that parthood and ϕ-parthood behave oddly ([10], p. 3)”.

The substantial issue between us seems to be the question whether intransitive and non-transitive ϕ-parthoods should be regarded as being binary or (at least) ternary. Obviously, as Varzi has now explicated ‘ϕ-parthood’, such parthoods may in principle have any kind of arity. As I will show, even when discussing this issue, it is important to keep terminological and substantial issues distinct.

2. According to my analysis: “Seemingly intransitive and non-transitive binary parthood predicates, both in everyday and in scientific language, are in every case hiding a reference to a third relatum ([6], p. 180)”. My point is that all relations denoted by truly *binary* parthood predicates are necessarily transitive, but that this is not true for all relations denoted by (explicitly or implicitly) *ternary* parthood predicates. In order to understand my view, one has to keep two well known but seldom used distinctions in mind: (1) the distinction between (1a) *truly monadic predicates* such as ‘is round’ and (1b) *relational predicates* such as ‘is as round as this circle’, and (2) the distinction between (2a) *truly binary relation predicates* such as ‘is parent of’ and (2b) *relative products* of binary relations such as ‘is grandparent of’. My four examples can be formalized as follows (note that R is used as a symbol for relations with different arity):

(1a) Rx (= ‘x is round’)
(1b) $\exists z(Rxz)$ (or Rxa) (= ‘x is as round as this circle’)

(2a) Rxy (= ‘x is parent of y’)
(2b) $R/Rxy =_{df} \exists z(Rxz \,\&\, Rzy)$ (= ‘x is grandparent of y’)

What is the arity of R in these four cases? With respect to (1a) and (2a) the answers are straightforward: in (1a) R is a unary predicate and in (2a) R is binary. But what about (1b) and (2b)? In these cases, one can distinguish between at least two kinds of arity for R: arity with respect only to *free* variables and arity with respect to *all* variables, respectively. It is unclear to me why, but Varzi wants to speak only of arity of the first kind, let us call it “free arity”, whereas in my paper I speak about arity of the second kind, “total arity”. Using this distinction, we can say that in (1b) R has a unary free arity but a binary total arity, and that in (2b) R has a binary free arity but a ternary total arity. The fact that in (2b) z is a bound variable is what makes z exist only implicitly in the definiendum R/Rxy. If in this definition we insert definite values in the variables (x = a, y = b, and z = c), it becomes quite obvious that R/R involves three relata, even though only two are explicitly mentioned in the definiendum and focussed on when the predicate is used in ordinary conversation. When we speak of a certain grandparent we note that there is a parent, too, but we put this person in the background. We get:

(2c) $R/Rab =_{df} (Rac \,\&\, Rcb)$.

Varzi writes in relation to (2b) that "the number of arguments of the definiendum [of relative products] is *perfectly* binary" ([10], p. 7, italics mine). Yes, but only in relation to free (and in definiendum explicit) arity. In relation to the total arity (which is partly implicit in the definiendum, but explicit in the definiens) it is perfectly ternary. In (2c), the number of individual constants is in the definiendum perfectly binary and in the definiens perfectly ternary. Since definiendum and definiens should have the same content, the individual constant c must be understood as being implicitly present in the definiendum of (2c), and the individual variable z must be understood as being implicitly present in the definiendum of (2b).

I cannot believe that Varzi wants to deny the obvious fact that relative products allow a distinction between two kinds of arity. Nor can I believe that he would like to deny that '∃z(Raz & Rzb)' better captures the formal structure of 'Ingvar is grandparent of Milan' than 'Gab' does. Therefore, if these two beliefs are true, there is with respect to relative products, as in the case of subsumption, no substantial disagreement between us. As soon as Varzi accepts my terminology for arity, he has also to accept my view that 'grandparent' can be regarded as a relation that has a *total* arity of three. That is, Varzi has to accept that where there are relative products there are relations that in the sense explained involve three relata.

The substantial issue then becomes whether there are any intransitive or non-transitive ϕ-parthoods that are relative products. Varzi says no, but he discusses only functional parthood (see section 4 below), and this is, I admit, a tricky case. But I would very much like to see an analysis of my most conspicuous example of non-transitive parthood, 'x is a large part of y', that does not (just like 'x is a grandparent of y') reveal some kind of relative product. To be large is to be large in comparison with something. Therefore, there must in the case at hand be an entity, z, distinct from x and y, in relation to which x is large. This example of non-transitivity has not been discussed earlier in the literature.

3. In [6], I present 15 examples of parthood predicates where transitivity does not seem to hold. I claim that two of these (6 and 15) lack transitivity because of equivocations, but that all the others lack transitivity because, appearances notwithstanding, they are not truly binary relations. That is, these ϕ-parthood predicates have an implicit reference to (at least) a third term. To repeat, I am firmly convinced that all truly binary parthood predicates are transitive. Varzi claims that my "third-and-hidden-relatum solution" is in all these thirteen cases either wrong or not really important (I am not sure how exactly to interpret him in this respect), and that, *instead*, transitivity "founders because" in these cases we have not simple parthood transitivity, 'Pxy & Pyz → Pxz', but (to quote him, [10], p. 8):

(10) (Pxy & Fxy) & (Pyz & Fyz) → (Pxz & Fxz)

Obviously, if Fxy is transitive then the ϕ-parthood 'Pxy & Fxy' is transitive, too. Varzi is not by means of (10) stating a *sufficient condition* for failing transitivity, only a formula that allows a conjunction of 'P' and 'F' to lack transitivity. If 'Fxy' = 'x is spatially smaller than y', then (10) yields the transitive predicate 'proper spatial parthood', but if 'Fxy' = 'x is 60% of y', then (10) yields the intransitive '60%-parthood'; and we ought to be able to say something about what makes these two 'Fxy's differ.

In formula (10), transitivity will hold or fail to hold depending on what particular relations are substituted for 'F', but in my paper I try to find exactly what makes transitivity fail. I claim that in such cases the definite relation in question is not a binary relation. Varzi says nothing, but his position requires that more should be said. In fact, my analysis can easily accomodate the predicate modifier account of non-transitive parthood (as I now understand it).

4. What then about *functional* parthood, which Varzi uses as an illustrative example when trying to show that intransitive parthood predicates do not involve relative products? I think that he takes too lightly my distinction between relative products and *qualified* relative products. Be this as it may, I need not to change any essential view of mine in admitting that functional parthood need not be analyzed as a relative product, so long as my view that the expression 'x is a functional part of y' ('Fxy') implicitly contains a third relatum, still stands. And in this respect Varzi is firmly on my side: "Johansson is right in pointing out that 'F' involves 'a hidden and indefinite reference to a third relatum' ([10], p. 7)". When Varzi's formulas (8) and (9) are put together we get formula (9'):

(9') $FPxy =_{df} Pxy \,\&\, \exists z(Mxyz)$

This formula should be read:

(9'') 'x is a Functional Part of y' $=_{df}$ 'x is Part of y' and 'there is a z such that x Makes something happen to z that is relevant for x's function in relation to y'

Which can be exemplified as:

(9''') 'the heart (x) is a Functional Part of the circulatory system (y)' $=_{df}$ 'the heart is Part of the circulatory system' and 'there is blood (z) such that the heart Makes something happen to it (z) that is relevant for the heart's function in relation to the circulatory system'

Except for the fact that I would like to have 'Pxy' mean spatial part instead of just part, the formulas (9'), (9''), and (9''') are in complete conformance with my general views on the intransitivity of functional parthood. Formula (9') takes us back to my proposed solution: the failure of transitivity is due to the fact that the relation under scrutiny ('Fxy') has a *total* arity of at least three (displayed in 'Mxyz'). My solution does not depend on whether or not functional parthood brings in a primitive ternary relation (as in the above example), a relative product in the traditional sense, or a qualified relative product (as I argued in my original paper). The important point is the fact that there is a third relatum involved; whether it is referred to by means of a free variable, a bound variable, or an individual name. In fact, I have recently published two papers specifically concerned with functions [7] [8] in which the notion of relative product is not used at all, but where, nonetheless, the claims made are explicitly in accordance with a three-term analysis of functions.

(In passing, Varzi's remark about wireless computer mice ([10], p. 6) is false. A mouse that cannot be connected to a certain computer by means of electromagnetic radiation is not a functional part of that computer.)

5. Next I will state a view of mine that directly take us back to my introductory question: what does the relation between classical formal mereology and the parthood predicates used in science and ordinary language look like? It concerns the efficiency of present-day ontological work in the information sciences. In my opinion, one should distinguish between 'formal-axiomatic mereology for *pure* mereology's sake' and 'formal-axiomatic mereology for *applied* mereology's sake'. Confronted with the formal-logical inter-definability of 'parthood' and 'proper parthood' present-day mereologists (but not Peter Simons in his classic *Parts* [9]) seem unanimously to opt for 'part' as their primitive undefined predicate. However, at least when it comes to ϕ-parthood, I think one should for pragmatic reasons choose 'proper ϕ-parthood' (e.g., 'proper spatial part', 'proper anatomical part' and 'proper functional part') as the undefined term. To take just one of an incredible number of possible examples, in medical information science one needs to say that ‘the heart is a proper part of the circulatory system’ but never that ‘the heart is *either* a proper part of *or* identical with the circulatory system’. It is confusing for non-mereologists that the mereological term that corresponds to our everyday and empirical-scientific term ‘part’ is ‘proper part’, not simply ‘part’. As far as I have checked, all ‘part_of’ occurrences in the GO and in the FMA can be correctly read as ‘proper part’; and it seems likely that this is also what the original authors meant.

6. For a long time I have liked the nuances of the following statement: “Physics is not applied mathematics. It is a natural science in which mathematics can be applied ([5], p. 72).” Of course, what is meant comes out better in the original context, but I hope that enough is conveyed in order to make the following paraphrase understandable:

- Ontology construction with parthood relations is not applied formal-axiomatic mereology. It is ontology construction in which, sometimes, formal-axiomatic mereology can be applied.

Acknowledgments

This reply was written under the auspices of the Wolfgang Paul Program of the Alexander von Humboldt Foundation, the Network of Excellence in Semantic Interoperability and Data Mining in Biomedicine of the European Union, and the project Forms of Life sponsored by the Volkswagen Foundation. I would like to thank Pierre Grenon, Fabian Neuhaus, and Carola Eschenbach for useful comments on an earlier version, and Andrew Spears and Nicola Guarino for comments on this one.

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